



Cambridge International AS & A Level

CANDIDATE
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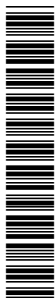
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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

October/November 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

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1 Solve the equation

$$\ln(1 + e^{-3x}) = 2.$$

Give the answer correct to 3 decimal places.

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- 2 (a) Expand $\sqrt[3]{1+6x}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]

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- (b) State the set of values of x for which the expansion is valid. [1]

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3 The variables x and y satisfy the relation $2^y = 3^{1-2x}$.

- (a) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. [3]

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- (b) Find the exact x -coordinate of the point of intersection of this line with the line $y = 3x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [2]

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4 (a) Show that the equation $\tan(\theta + 60^\circ) = 2 \cot \theta$ can be written in the form

$$\tan^2 \theta + 3\sqrt{3} \tan \theta - 2 = 0. \quad [3]$$

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- (b) Hence solve the equation $\tan(\theta + 60^\circ) = 2 \cot \theta$, for $0^\circ < \theta < 180^\circ$. [3]

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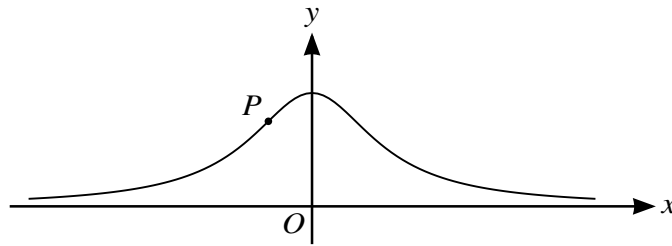
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The diagram shows the curve with parametric equations

$$x = \tan \theta, \quad y = \cos^2 \theta,$$

for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

- (a) Show that the gradient of the curve at the point with parameter θ is $-2 \sin \theta \cos^3 \theta$. [3]

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6 The complex number u is defined by

$$u = \frac{7+i}{1-i}.$$

(a) Express u in the form $x + iy$, where x and y are real. [3]

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(b) Show on a sketch of an Argand diagram the points A , B and C representing u , $7 + i$ and $1 - i$ respectively. [2]

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(c) By considering the arguments of $7 + i$ and $1 - i$, show that

$$\tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{1}{7}\right) + \frac{1}{4}\pi. \quad [3]$$

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7 The variables x and t satisfy the differential equation

$$e^{3t} \frac{dx}{dt} = \cos^2 2x,$$

for $t \geq 0$. It is given that $x = 0$ when $t = 0$.

(a) Solve the differential equation and obtain an expression for x in terms of t . [7]

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(b) State what happens to the value of x when t tends to infinity. [1]

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- 8 With respect to the origin O , the position vectors of the points A , B , C and D are given by

$$\vec{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \quad \vec{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \vec{OD} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

- (a) Show that $AB = 2CD$.

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- (b) Find the angle between the directions of \vec{AB} and \vec{CD} .

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9 Let $f(x) = \frac{7x + 18}{(3x + 2)(x^2 + 4)}$.

(a) Express $f(x)$ in partial fractions. [5]

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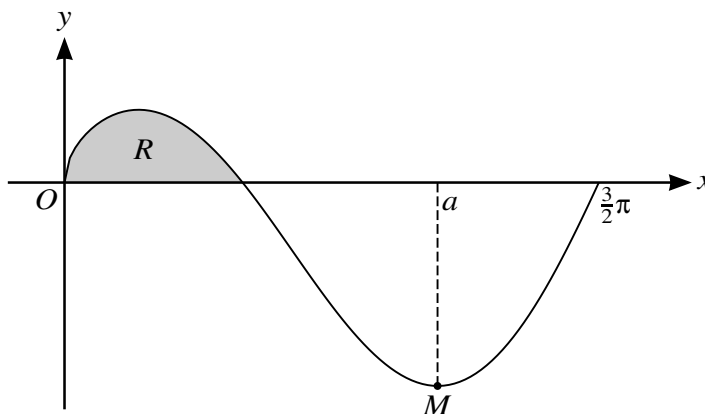
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The diagram shows the curve $y = \sqrt{x} \cos x$, for $0 \leq x \leq \frac{3}{2}\pi$, and its minimum point M , where $x = a$. The shaded region between the curve and the x -axis is denoted by R .

- (a) Show that a satisfies the equation $\tan a = \frac{1}{2a}$. [3]

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- (b) The sequence of values given by the iterative formula $a_{n+1} = \pi + \tan^{-1}\left(\frac{1}{2a_n}\right)$, with initial value $x_1 = 3$, converges to a .

Use this formula to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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- (c) Find the volume of the solid obtained when the region R is rotated completely about the x -axis. Give your answer in terms of π . [6]

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